KUROSH-AMITSUR RADICALS OF GROUPS: SOMETHING FOR EVERYONE?

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Abstract. The usual account of the genesis of abstract radical theory begins with rings, and mentions other algebraic structure only in the context of developments subsequent to the publication of the fundamental papers of Amitsur and Kurosh in the 1950s. However, an examination of the mathematical literature of the USSR reveals that group theory exerted a major influence on the development of the abstract radical concept: a striking example of this sort of thing is the first use of transfinite normal series accessible-in-themselves (later used in the treatment of radicals in rings and algebras) in a 1935 paper of Kurosh on a generalization of the Jordan-Hölder Theorem for infinite groups. Radical theory has developed rather differently for rings and groups, and it is worthwhile considering this in the light of "cultural" differences. The "big questions" tend to have the same answers for groups and associative rings (and sometimes with easier proofs in the group case) but the lower radical construction, though it never needs more than ω steps for groups, has never been shown to need more than 3. There is an abundance of results on the behaviour of concrete radicals on restricted classes of groups. While radical theory for groups went quiet for a while, in recent years a number of workers in group theory and algebraic topology have looked at various specific radicals with, it seems, limited awareness of a radical theoretic tradition within which their examples fit and in some cases have been investigated. All of these topics will be discussed in this survey.