On supernilpotent nonspecial radicals

Halina France-Jackson

Department of Mathematics and Applied Mathematics
Summerstrand Campus (South)
PO Box 77000
Nelson Mandela Metropolitan University
Port Elizabeth 6031
South Africa
e-mail: cbf@easterncape.co.uk

Abstract

All rings in this paper are associative and all classes of rings contain the one-element ring 0. All undefined radical theoretic terms and facts can be found in [1] and [4]. An ideal I of a ring A is called essential if $I \cap J \neq 0$ whenever J is a nonzero ideal of A. A class μ of rings is hereditary if μ is closed under ideals. A hereditary class μ of semiprime rings is called weakly special if μ is essentially closed, that is, whenever $I \in \mu$ is an essential ideal of a ring A, then $A \in \mu$ also holds. Throughout this paper, for a class μ of rings, $\mathcal{U}(\mu)$ will denote the class of all rings which have no nonzero homomorphic image in μ, $\mathcal{L}(\mu)$ will denote the lower radical class determined by μ and $\mathcal{S}(\mu)$ will stand for the class of all rings without nonzero ideals in μ. Moreover, μ^* [5] will denote the class of all rings A such that either A is a simple ring in μ or the factor ring A/I is in μ for every nonzero ideal I of A and every minimal ideal M of A is in μ. A supernilpotent radical is a hereditary radical class which contains all nilpotent rings. It is well known [1], [4] that ρ is a supernilpotent radical if and only if $\rho = \mathcal{U}(\mu)$ for some weakly special class μ of rings. In [5], H. J. Le Roux and G. A. P. Heyman proved that if ρ is a supernilpotent radical, then so is $\mathcal{L}(\rho^*)$ and $\rho \subseteq \mathcal{L}(\rho^*) \subseteq \rho_\varphi$, where ρ_φ denotes the upper radical determined by the class of all subdirectly irreducible rings with ρ-semisimple hearts. Moreover, $\mathcal{L}(\mathcal{G}^*) = \mathcal{G}_\varphi$, where $\mathcal{G}$ is the Brown-McCoy radical. They asked whether $\mathcal{L}(\rho^*) = \rho_\varphi$ if ρ is replaced by β, $\mathcal{L}$, $\mathcal{N}$ or $\mathcal{J}$, where β, $\mathcal{L}$, $\mathcal{N}$ and $\mathcal{J}$ denote the Baer, the Levitzki, the Koethe and the Jacobson radical, respectively. In the present paper we will give a negative answer to this question.

References

[1] N. Divinsky, *Rings and radicals*, Allen & Unwin, London, 1965.

[2] H. France-Jackson, *-rings and their radicals, *Quaest. Math.* 8 (1985), no. 3, 231-239.

[3] B. J. Gardner and P. N. Stewart, Prime essential rings, *Proc. Edinburgh Math. Soc.* 34 (1991), 241-250.

[4] B. J. Gardner and R. Wiegandt, *Radical theory of rings*, Marcel Dekker, New York, 2004.

[5] H. J. Le Roux and G. A. P. Heyman, A question on the characterization of certain upper radical classes, *Bollettino U. M. I.* (5) 17-A (1980), 67-72.